Contents

Acknowledgments xiii

Chapter 1 Introduction 1
1.1 Types of Disease 1
1.2 Characterization of Diseases 3
1.3 Control of Infectious Diseases 5
1.4 What Are Mathematical Models? 7
1.5 What Models Can Do 8
1.6 What Models Cannot Do 10
1.7 What Is a Good Model? 10
1.8 Layout of This Book 11
1.9 What Else Should You Know? 13

Chapter 2 Introduction to Simple Epidemic Models 15
2.1 Formulating the Deterministic SIR Model 16
  2.1.1 The SIR Model Without Demography 19
    2.1.1.1 The Threshold Phenomenon 19
    2.1.1.2 Epidemic Burnout 21
    2.1.1.3 Worked Example: Influenza in a Boarding School 26
  2.1.2 The SIR Model With Demography 26
    2.1.2.1 The Equilibrium State 28
    2.1.2.2 Stability Properties 29
    2.1.2.3 Oscillatory Dynamics 30
    2.1.2.4 Mean Age at Infection 31
2.2 Infection-Induced Mortality and SIS Models 34
  2.2.1 Mortality Throughout Infection 34
    2.2.1.1 Density-Dependent Transmission 35
    2.2.1.2 Frequency Dependent Transmission 36
  2.2.2 Mortality Late in Infection 37
  2.2.3 Fatal Infections 38
2.3 Without Immunity: The SIS Model 39
2.4 Waning Immunity: The SIRS Model 40
2.5 Adding a Latent Period: The SEIR Model 41
2.6 Infections with a Carrier State 44
2.7 Discrete-Time Models 46
2.8 Parameterization 48
  2.8.1 Estimating $R_0$ from Reported Cases 50
  2.8.2 Estimating $R_0$ from Seroprevalence Data 51
  2.8.3 Estimating Parameters in General 52
2.9 Summary 52
## Chapter 3 Host Heterogeneities

### 3.1 Risk-Structure: Sexually Transmitted Infections
- **3.1.1 Modeling Risk Structure**
  - 3.1.1.1 High-Risk and Low-Risk Groups
  - 3.1.1.2 Initial Dynamics
  - 3.1.1.3 Equilibrium Prevalence
  - 3.1.1.4 Targeted Control
  - 3.1.1.5 Generalizing the Model
  - 3.1.1.6 Parameterization
- **3.1.2 Two Applications of Risk Structure**
  - 3.1.2.1 Early Dynamics of HIV
  - 3.1.2.2 Chlamydia Infections in Koalas
- **3.1.3 Other Types of Risk Structure**

### 3.2 Age-Structure: Childhood Infections
- **3.2.1 Basic Methodology**
  - 3.2.1.1 Initial Dynamics
  - 3.2.1.2 Equilibrium Prevalence
  - 3.2.1.3 Control by Vaccination
  - 3.2.1.4 Parameterization
- **3.2.2 Applications of Age Structure**
  - 3.2.2.1 Dynamics of Measles
  - 3.2.2.2 Spread and Control of BSE
- **3.3 Dependence on Time Since Infection**
  - 3.3.1 SEIR and Multi-Compartment Models
  - 3.3.2 Models with Memory
  - 3.3.3 Application: SARS
- **3.4 Future Directions**

### 3.5 Summary

## Chapter 4 Multi-Pathogen/Multi-Host Models

### 4.1 Multiple Pathogens
- **4.1.1 Complete Cross-Immunity**
  - 4.1.1.1 Evolutionary Implications
- **4.1.2 No Cross-Immunity**
  - 4.1.2.1 Application: The Interaction of Measles and Whooping Cough
  - 4.1.2.2 Application: Multiple Malaria Strains
- **4.1.3 Enhanced Susceptibility**
- **4.1.4 Partial Cross-Immunity**
  - 4.1.4.1 Evolutionary Implications
  - 4.1.4.2 Oscillations Driven by Cross-Immunity
- **4.1.5 A General Framework**

### 4.2 Multiple Hosts
- **4.2.1 Shared Hosts**
  - 4.2.1.1 Application: Transmission of Foot-and-Mouth Disease
  - 4.2.1.2 Application: Parapoxvirus and the Decline of the Red Squirrel
<table>
<thead>
<tr>
<th>Page</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>54</td>
<td>4.2.2 Vectored Transmission</td>
</tr>
<tr>
<td>55</td>
<td>4.2.2.1 Mosquito Vectors</td>
</tr>
<tr>
<td>57</td>
<td>4.2.2.2 Sessile Vectors</td>
</tr>
<tr>
<td>57</td>
<td>4.2.3 Zoonoses</td>
</tr>
<tr>
<td>59</td>
<td>4.2.3.1 Directly Transmitted Zoonoses</td>
</tr>
<tr>
<td>62</td>
<td>4.2.3.2 Vector-Borne Zoonoses: West Nile Virus</td>
</tr>
<tr>
<td>63</td>
<td>4.3 Future Directions</td>
</tr>
<tr>
<td>64</td>
<td>4.4 Summary</td>
</tr>
<tr>
<td>69</td>
<td>Chapter 5 Temporally Forced Models</td>
</tr>
<tr>
<td>71</td>
<td>5.1 Historical Background</td>
</tr>
<tr>
<td>74</td>
<td>5.1.1 Seasonality in Other Systems</td>
</tr>
<tr>
<td>76</td>
<td>5.2 Modeling Forcing in Childhood Infectious Diseases: Measles</td>
</tr>
<tr>
<td>77</td>
<td>5.2.1 Dynamical Consequences of Seasonality: Harmonic and Subharmonic Resonance</td>
</tr>
<tr>
<td>78</td>
<td>5.2.2 Mechanisms of Multi-Annual Cycles</td>
</tr>
<tr>
<td>80</td>
<td>5.2.3 Bifurcation Diagrams</td>
</tr>
<tr>
<td>81</td>
<td>5.2.4 Multiple Attractors and Their Basins</td>
</tr>
<tr>
<td>82</td>
<td>5.2.5 Which Forcing Function?</td>
</tr>
<tr>
<td>84</td>
<td>5.2.6 Dynamical Transitions in Seasonally Forced Systems</td>
</tr>
<tr>
<td>89</td>
<td>5.3 Seasonality in Other Diseases</td>
</tr>
<tr>
<td>93</td>
<td>5.3.1 Other Childhood Infections</td>
</tr>
<tr>
<td>94</td>
<td>5.3.2 Seasonality in Wildlife Populations</td>
</tr>
<tr>
<td>98</td>
<td>5.3.2.1 Seasonal Births</td>
</tr>
<tr>
<td>100</td>
<td>5.3.2.2 Application: Rabbit Hemorrhagic Disease</td>
</tr>
<tr>
<td>102</td>
<td>5.4 Summary</td>
</tr>
<tr>
<td>103</td>
<td>Chapter 6 Stochastic Dynamics</td>
</tr>
<tr>
<td>105</td>
<td>6.1 Observational Noise</td>
</tr>
<tr>
<td>106</td>
<td>6.2 Process Noise</td>
</tr>
<tr>
<td>107</td>
<td>6.2.1 Constant Noise</td>
</tr>
<tr>
<td>109</td>
<td>6.2.2 Scaled Noise</td>
</tr>
<tr>
<td>112</td>
<td>6.2.3 Random Parameters</td>
</tr>
<tr>
<td>112</td>
<td>6.2.4 Summary</td>
</tr>
<tr>
<td>115</td>
<td>6.2.4.1 Contrasting Types of Noise</td>
</tr>
<tr>
<td>116</td>
<td>6.2.4.2 Advantages and Disadvantages</td>
</tr>
<tr>
<td>125</td>
<td>6.3 Event-Driven Approaches</td>
</tr>
<tr>
<td>128</td>
<td>6.3.1 Basic Methodology</td>
</tr>
<tr>
<td>130</td>
<td>6.3.1.1 The SIS Model</td>
</tr>
<tr>
<td>131</td>
<td>6.3.2 The General Approach</td>
</tr>
<tr>
<td>133</td>
<td>6.3.2.1 Simulation Time</td>
</tr>
<tr>
<td></td>
<td>6.3.3 Stochastic Extinctions and The Critical Community Size</td>
</tr>
<tr>
<td></td>
<td>6.3.3.1 The Importance of Imports</td>
</tr>
<tr>
<td></td>
<td>6.3.3.2 Measures of Persistence</td>
</tr>
<tr>
<td></td>
<td>6.3.3.3 Vaccination in a Stochastic Environment</td>
</tr>
<tr>
<td></td>
<td>6.3.4 Application: Porcine Reproductive and Respiratory Syndrome</td>
</tr>
<tr>
<td></td>
<td>6.3.5 Individual-Based Models</td>
</tr>
</tbody>
</table>
6.4 Parameterization of Stochastic Models
6.5 Interaction of Noise with Heterogeneities
  6.5.1 Temporal Forcing
  6.5.2 Risk Structure
  6.5.3 Spatial Structure
6.6 Analytical Methods
  6.6.1 Fokker-Plank Equations
  6.6.2 Master Equations
  6.6.3 Moment Equations
6.7 Future Directions
6.8 Summary

Chapter 7 Spatial Models
7.1 Concepts
  7.1.1 Heterogeneity
  7.1.2 Interaction
  7.1.3 Isolation
  7.1.4 Localized Extinction
  7.1.5 Scale
7.2 Metapopulations
  7.2.1 Types of Interaction
    7.2.1.1 Plants
    7.2.1.2 Animals
    7.2.1.3 Humans
    7.2.1.4 Commuter Approximations
  7.2.2 Coupling and Synchrony
  7.2.3 Extinction and Rescue Effects
  7.2.4 Levins-Type Metapopulations
  7.2.5 Application to the Spread of Wildlife Infections
    7.2.5.1 Phocine Distemper Virus
    7.2.5.2 Rabies in Raccoons
7.3 Lattice-Based Models
  7.3.1 Coupled Lattice Models
  7.3.2 Cellular Automata
    7.3.2.1 The Contact Process
    7.3.2.2 The Forest-Fire Model
    7.3.2.3 Application: Power laws in Childhood Epidemic Data
7.4 Continuous-Space Continuous-Population Models
  7.4.1 Reaction-Diffusion Equations
  7.4.2 Integro-Differential Equations
7.5 Individual-Based Models
  7.5.1 Application: Spatial Spread of Citrus Tristeza Virus
  7.5.2 Application: Spread of Foot-and-mouth Disease in the United Kingdom
7.6 Networks
  7.6.1 Network Types
    7.6.1.1 Random Networks
    7.6.1.2 Lattices

## CONTENTS

219 7.6.1.3 Small World Networks 279
219 7.6.1.4 Spatial Networks 279
219 7.6.1.5 Scale-Free Networks 279
220 7.6.2 Simulation of Epidemics on Networks 280
221 7.7 Which Model to Use? 282
222 7.8 Approximations 283
222 7.8.1 Pair-Wise Models for Networks 283
223 7.8.2 Pair-Wise Models for Spatial Processes 286
227 7.9 Future Directions 287
230 7.10 Summary 288

### Chapter 8 Controlling Infectious Diseases

232 8.1 Vaccination 291
233 8.1.1 Pediatric Vaccination 292
233 8.1.2 Wildlife Vaccination 296
235 8.1.3 Random Mass Vaccination 297
236 8.1.4 Imperfect Vaccines and Boosting 298
236 8.1.5 Pulse Vaccination 301
236 8.1.6 Age-Structured Vaccination 303
240 8.1.6.1 Application: Rubella Vaccination 304
240 8.1.7 Targeted Vaccination 306
240 8.2 Contact Tracing and Isolation 308
241 8.2.1 Simple Isolation 309
242 8.2.2 Contact Tracing to Find Infection 312
243 8.3 Case Study: Smallpox, Contact Tracing, and Isolation 313
245 8.4 Case Study: Foot-and-Mouth Disease, Spatial Spread, and Local Control 321
246 8.5 Case Study: Swine Fever Virus, Seasonal Dynamics, and Pulsed Control 327
250 8.5.1 Equilibrium Properties 329
251 8.5.2 Dynamical Properties 331
252 8.6 Future Directions 333
252 8.7 Summary 334

References 337

Index 361

Parameter Glossary 367